Approximators as a case study of attenuating polarity items[1]

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1 Introduction

Approximators such as *about* exhibit a curious and little-recognized form of polarity sensitivity: in simple sentences they are positive polarity items (PPIs), whereas when embedded in comparative quantifiers, they become negative polarity items (NPIs):

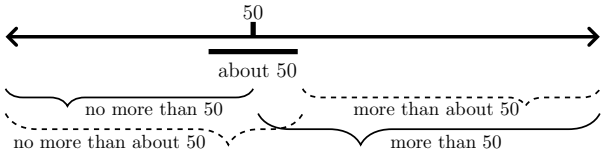
(1) a. Lisa has / *doesn't have about 50 sheep.
b. Lisa *has / doesn't have more than about 50 sheep.
- Here we ignore the echoic use.

(See Israel 2006; Rodríguez 2008; Spector 2014 for some discussion.)

In their felicitous occurrences, approximator-modified expressions make weaker assertions than their unmodified counterparts. As such, they belong to the class of **attenuating** polarity items (Israel 1996, 2006, 2011); this is also evidenced by their hedging feel.

(2) a. Lisa has 50 sheep ⇒ Lisa has about 50 sheep.
b. Lisa doesn't have more than 50 sheep ⇒ Lisa doesn't have more than about 50 sheep.

(3) Ranges corresponding to numerical expressions:



Attenuators – other classic examples of which are illustrated below – have received little attention from the perspective of formal semantics, and there is no existing account of the pattern in (1).

(4) Homer *slept much / didn't sleep much.

(5) Bart is / *isn't fairly lazy.

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Goal: The goal of the present work is to develop a formal semantic/pragmatic account of the polarity sensitivity of approximators, as an initial step towards a more general theory of the attenuating class.

Core ideas in a nutshell:

- Approximators, like other polarity items, obligatorily introduce alternatives. Since they are modifiers, their alternatives necessarily include the corresponding unmodified form.
- Being vague/lacking sharp boundaries, approximators do not readily make stronger statements than salient alternatives – notably the unmodified one. They are thus restricted to situations in which the simpler unmodified form could not have been asserted.

2 A neo-Gricean framework

I implement the analysis in a neo-Gricean framework based on Katzir (2007) and more indirectly Krifka (1995), whose central component is the following principle:

(6) **Conversational principle:** Do not use ϕ if there is another sentence $\phi' \in ALT(\phi)$ such that both:
i. ϕ' is better than ϕ ($\phi \succ \phi'$)
ii. ϕ' is weakly assertable

This principle has the following consequences:

- In asserting ϕ, the speaker implicates that all better alternatives ϕ' cannot be weakly asserted.
- Polarity-based restrictions arise when an implicature derived in this way contradicts the original assertion, or equivalently, when a given expression always has a better alternative

We assume the following definitions:

- Alternatives are derived via substitution and deletion (Katzir 2007):

(7) **Substitution source:**
Let ϕ be a parse tree. The substitution source for ϕ, written as $L(\phi)$, is the lexicon of the language.

(8) **Structural alternatives:**
Let ϕ be a parse tree. $ALT(\phi)$ – the set of alternatives to ϕ – is the set of parse trees ϕ' that can be derived from ϕ via a finite series of deletions, contractions, and replacements of constituents in ϕ with constituents of the same category taken from $L(\phi)$.

- A sentence is weakly assertable if the speaker believes it to be true, relevant and supported by the evidence.

Departing somewhat from Katzir, I assume the 'better than' relation to be defined in terms of both **informativity** and **structural simplicity** (cf. the maxims of Quantity and Manner of Grice 1975):

(9) $\phi \succ \psi$ iff $\phi \succsim_{INF} \psi \wedge \phi \succsim_{SIMP} \psi \wedge (\phi \succ_{INF} \psi \vee \phi \succ_{SIMP} \psi)$

- ϕ is better than ψ if it is at least as informative and at least as simple, and has an advantage on either informativity or simplicity.

Turning to the component parts of the 'better than' relation, simplicity can be defined in structural terms:

(10) **Simplicity**:
a. $\phi \succsim_{SIMP} \psi$ iff ϕ can be derived from ψ via substitution/deletion
b. $\phi \succ_{SIMP} \psi$ iff $\phi \succsim_{SIMP} \psi$ and not $\psi \succsim_{SIMP} \phi$

Semantic strength or informativity is most commonly understood in terms of unidirectional entailment:

(11) **Informativity:**
a. $\phi \succsim_{INF} \psi$ iff $[\![\phi]\!] \subseteq [\![\psi]\!]$
b. $\phi \succ_{INF} \psi$ iff $\phi \succsim_{INF} \psi$ and not $\psi \succsim_{INF} \phi$

I propose however that this must be modified, in particular to account for vagueness. Suppose sentence ϕ has an alternative ϕ' derived by replacing some element α with a suitable element α' (where α' may be null).

- If both α and α' are non-vague, determining the relative strength of ϕ and ϕ' is straightforward (examples: *or* vs. *and*; *some* vs. *all*).

- However, if α and/or α' is vague or context-sensitive – or perhaps more accurately, if α and α' form in the context of ϕ and ϕ' the predicates P and P', one or both of which is vague/context-sensitive – then relative strength is less clear.

The intuition that I pursue is that for the purpose of comparing alternatives, the 'more informative than' relation $\succ_{INF}$ must be interpreted as 'significantly' or 'clearly' more informative than. Evidence for such a principle comes from the domain of scalar implicatures:

- van Tiel et al. (2016) demonstrate that scalar implicatures are more likely to arise between pairs of scalar adjectives when the scalar distance between them is greater (e.g. *difficult* tends to implicate *not impossible*, whereas *adequate* does not consistently implicate *not good*).

- Leffel et al. (2016) note an asymmetry in the interpretation of *not very Adj*: with minimum-standard absolute gradable adjectives, this licenses the inference that the positive form *Adj* holds, while with context-sensitive relative gradable adjectives, no such inference can be drawn:

(12) a. John was not very late. $\rightsquigarrow$ John was late.
b. The antenna is not very bent. $\rightsquigarrow$ The antenna is bent.

(13) a. John is not very tall. $\not\rightsquigarrow$ John is tall.
b. John isn't very smart. $\not\rightsquigarrow$ John is smart.

Put differently, the examples in (12) license the inference to the negation of the simpler form *not Adj* – i.e. a scalar implicature – while the examples in (13) do not. Leffel and colleagues account for this pattern via a constraint on the derivation of implicatures with vague predicates, which draws on the notion of 'borderline contradictions' (Alxatib and Pelletier 2011; Ripley 2011):

(14) **Constraint on vague implicatures**
If a sentence S has an alternative S', the potential implicature $\neg S'$ is not drawn if $S \wedge \neg S'$ would necessarily be a *borderline contradiction*.

(15) If F, G are gradable predicates, $Fa \wedge \neg Ga$ is a borderline contradiction iff a is a borderline case for both F and G.

– Intuitively, there are situations that are simultaneously clear cases of both *late* and *not very late*, but any individual that is *tall and not very tall* is necessarily a borderline case of both individual predicates – i.e. the vague standards of *tall* and *very tall* are not sufficiently distinct for an implicature to be drawn from *not very tall* to *NOT(not tall)=tall*.

Building on the analysis of Leffel et al., I thus propose the following revised notion of informativity:

(16) **Informativity (revised):**
$\phi \succ_{INF} \psi$ iff in any context (i.e. any way of filling in contextual variables), there are states of affairs that are clear cases of both ψ and $NOT\ \phi$.

3 Analysis of approximators

3.1 Preliminaries

I follow Kennedy (2015) in taking numerals to have type-flexible degree-based interpretations:

(17) a. $[\![\textit{fifty}_d]\!] = 50$
b. $[\![\textit{fifty}_{\langle dt,t\rangle}]\!] = \lambda D_{dt}.max(D) = 50$

I further take approximator-modified numerals to be interpreted in terms of coarse-grained degrees, i.e. scalar intervals understood as unitary wholes:[2]

(18) a. $[\![\textit{about fifty}_d]\!] = 50_{COARSE}$
b. $[\![\textit{about fifty}_{\langle dt,t\rangle}]\!] = \lambda D_{dt}.max(D) = 50_{COARSE}$,
where $50_{COARSE} = 50 \pm k_c$, for some contextually determined k_c

Finally, comparative modified numerals are likewise analyzed as quantifiers over degrees:

(19) a. $[\![\text{more than}]\!] = \lambda d_d \lambda D_{dt}.max(D) > d$
b. $[\![\text{more than } 50]\!] = \lambda D_{dt}.max(D) > 50$

3.2 About *about*

We begin with the basic example of an approximator in a simple positive sentence (20), which minimally competes with the alternative (21) derived via deletion of the approximator:

(20) Lisa has about 50 sheep.
$max\{n : \text{Lisa has } n \text{ sheep}\} = 50_{COARSE} = 50 \pm k_c$

(21) Lisa has 50 sheep.
$max\{n : \text{Lisa has } n \text{ sheep}\} = 50$

Note the following:

- The denotation of the approximator-modified numeral corresponds to a contextually determined interval around the precise point-based denotation of the numeral, which in the limiting case is just that point (interval=point).
- Additionally, round numbers themselves allow approximate or coarse-grained interpretations (Krifka 2007); that is, instead of 50 we might more correctly write 50_{gran}.

Thus it is not the case that in any context, there is always a value that is both a clear case of *about 50* while simultaneously being a clear case of *NOT 50*. By the definition in (16), the alternatives in (20) and (21) are thus formally equivalent in informativity. Factoring in simplicity, we derive the following comparisons:

(22) ϕ = *Lisa has about 50 sheep* ϕ' = *Lisa has 50 sheep*
$\phi \sim_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi' \succ \phi$

[2] In Kennedy's system, the type d interpretation of *about 50* in (18a) would yield a lower-bounded 'at least' reading. According to my intuitions this interpretation is not available; i.e. *Lisa has about 50 sheep* cannot mean that she has about 50 or perhaps more. I leave it as an open question whether this reading is in fact semantically available but undetectable (as in the phantom readings of *between* constructions identified by Marty et al. 2014), or whether *about* constructions in simple sentences always have an interpretation based on the degree-quantifier entry in (18b).

(23) ϕ = *Lisa doesn't have about 50 sheep* ϕ' = *Lisa doesn't have 50 sheep*
$\phi \sim_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi' \succ \phi$

In both cases, the unmodified alternative is better than the original sentence. The assertion of the approximator-modified version thus yields the implicature that the unmodified utterance is not weakly assertable.

(24) *Lisa has / doesn't have about 50 sheep* $\rightsquigarrow$
Lisa has / doesn't have 50 sheep is not weakly assertable

Crucially, the effect of this is different in positive and negative examples. We assume that in a context in which an approximator such as *about* is used, the corresponding unmodified numeral is interpreted precisely.

- In the positive case (22), we derive the hedging or attenuating effect: the assertion of *about* 50 implicates that the speaker is not in the position to assert 50_{EXACT} (i.e. does not believe it to be true / relevant / supported).
- In the negative case (23), however, the implicature systematically contradicts the assertion: because $50_{EXACT} \subset 50_{COARSE}$, any speaker in a position to assert *not about* 50 is necessarily also in a position to assert its better alternative *not* 50_{EXACT}. **Thus the original sentence is blocked**.

When numerical expressions are embedded in comparative quantifiers, the situation is reversed. Again, the alternative without the approximator is the better one:

(25) ϕ = *Lisa has / doesn't have more than about 50 sheep*
ϕ' = *Lisa has / doesn't have more than 50 sheep*
$\phi \sim_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi' \succ \phi$

- In the negative case, the assertion of *not more than about* 50 yields the implicature that the speaker is not in a position to assert *not more than* 50_{EXACT}.
- In the positive case, the assertion of *more than about* 50 is blocked by its better alternative *not more than* 50_{EXACT}, which is also necessarily assertable.

A prediction of this analysis is that approximators will be acceptable in all and only constructions where the denotation of the bare numeral is properly contained in the denotation of the modified numeral construction. This prediction is borne out:

(26) a. Lisa has at most about 50 sheep.
b. Most farmers have between about 50 and about 200 sheep.
c. *Lisa has over about 50 sheep.

Finally, note that to account for the polarity sensitivity of approximators, it has been sufficient to consider only a single alternative of an approximator-modified numeral, namely the one derived via deletion of the approximator. The structural view actually generates further alternatives, giving rise to additional implicatures. This correctly captures other interpretive facts with approximator-modified numerals, e.g. that *no more than about 50* in some contexts implicates *about 50*.

(27) a. ϕ = Lisa doesn't have more than about 50 sheep
b. $ALT(\phi)$ = {Lisa doesn't have more than about n sheep : $n = \ldots 40, 50, 60, \ldots\}$ $\cup$ {Lisa doesn't have more than n sheep : $n = \ldots 40, 50, 60, \ldots\}$

(28) ϕ = *Lisa doesn't have more than about 50 sheep*
ϕ' = *Lisa doesn't have more than about 40 sheep*

$\phi' \succ_{INF} \phi$ $\quad$ $\phi' \sim_{SIMP} \phi$ $\quad$ $\phi' \succ \phi$

$\rightsquigarrow$ The speaker believes that it's not the case that Lisa doesn't have more than about 40 sheep, i.e. believes that she has about 50.

3.3 Other approximating constructions

The pattern of polarity sensitivity investigated here is observed with a range of approximators and approximating constructions:

(29) a. Lisa has roughly 50 / approximately 50 / 40 or 50 sheep.
b. *Lisa doesn't have roughly 50 / approximately 50 / 40 or 50 sheep.

(30) a. * Lisa has more than roughly 50 / approximately 50 / 40 or 50 sheep.
b. Lisa doesn't have more than roughly 50 / approximately 50 / 40 or 50 sheep.

However, approximators diverge in the sorts of implicatures they give rise to:

- As noted above, the present framework generates for *about n* a weak implicature that the speaker doesn't believe that *exactly n* obtains, but crucially does not generate a true ignorance implicature (i.e. an implicature that the speaker neither believes that *exactly n* obtains nor believes that it doesn't obtain).

  (31) Lisa has about 50 sheep $\rightsquigarrow$ the speaker doesn't hold the belief that Lisa has exactly 50 sheep.

  This correctly captures the facts: (31) could be asserted by a speaker who knows Lisa has (say) 47 sheep but chooses to round off to the more salient value 50; however, it is less felicitous if the speaker knows that *exactly 50* obtains.

- The rounding use is possible for a range of approximators:

  (32) About / roughly / around / approximately 70 patients took part in the study.

- Other approximating constructions however have a stronger ignorance effect:

  (33) Lisa has 40 or 50 sheep $\rightsquigarrow$ the speaker doesn't know the exact number.
  (34) ?? 60 or 70 patients took part in the study.

The present framework derives this ignorance effect in a similar way to other accounts of ignorance effects with disjunctions (e.g. Katzir 2007).

Approximating disjunctions of the form in (33) have the two individual disjuncts as alternatives. As described in Solt (2016), I take the use of such disjunctions to establish that the contextual level of granularity at which numerical expressions are interpreted is equal to the gap between the two values.

(35) ϕ = Lisa has 40 or 50 sheep. $\quad$ $max\{n : \text{Lisa has } n \text{ sheep}\} = 40_{gran=10} \cup 50_{gran=10}$
ϕ' = Lisa has 40 sheep. $\quad$ $max\{n : \text{Lisa has } n \text{ sheep}\} = 40_{gran=10}$
ϕ'' = Lisa has 50 sheep. $\quad$ $max\{n : \text{Lisa has } n \text{ sheep}\} = 50_{gran=10}$

Both alternatives ϕ' and ϕ'' are better than the original sentence ϕ, being both more informative and simpler. We thus derive the following implicatures, which amount to an ignorance effect:

(36) a. $\rightsquigarrow$ The speaker doesn't hold the belief that $40_{gran=10}$ obtains ($\equiv$ the speaker doesn't hold the belief that $50_{gran=10}$ doesn't obtain)
b. $\rightsquigarrow$ The speaker doesn't hold the belief that $50_{gran=10}$ obtains ($\equiv$ the speaker doesn't hold the belief that $40_{gran=10}$ doesn't obtain)

4 Other analytical approaches

In this section, I briefly consider how the approximator facts might be accounted for within a grammatical theory of implicatures and polarity sensitivity.

4.1 Silent *only*

Following Chierchia (2004, 2006, 2013), we might propose that polarity sensitivity derives from the presence of an operator O, a silent counterpart of overt *only*.

(37) $[\![O]\!] = \lambda p \lambda w.[p_w \wedge \forall q \in ALT(p)[q_w \rightarrow p \subseteq q]]$

- Has the effect of negating all non-entailed alternatives.

However, a straightforward implementation of this approach yields the wrong results for approximators:

(38) p=Lisa has about 50 sheep.
$ALT(p)$ = {Lisa has n sheep : $n \in 50_{COARSE}$}

(39) O(Lisa has 50 sheep)

- Contradiction! The alternatives exhaust the scalar range corresponding to *about 50*. Sentence predicted to be ungrammatical.
- If such alternatives are excluded via Innocent Exclusion (Fox 2007), no implicatures arise.

(40) p=Lisa doesn't have about 50 sheep.
$ALT(p) = \{$Lisa doesn't have n sheep : $n \in 50_{COARSE}\}$

(41) O(Lisa doesn't have 50 sheep)

- All alternatives are entailed; sentence predicted to be fine.

Thus a (probably overly) simple implementation of this approach accounts for neither the polarity sensitivity of approximators nor the implicatures that arise in their felicitous uses.

- A solution in terms of scale truncation (per Chierchia 2013 on *much*) does not readily extend to approximators.

4.2 Obligatory exhaustification

More promising results are obtained by incorporating some more recent proposals, in particular the notion of obligatory exhaustification (Spector 2014, 2015) as well as a covert doxastic operator K Meyer (2013).

(42) $exh(p) = p \wedge \forall q \in IE(p; Alt(p)) : \neg q$.

- An alternative is innocently excludable (IE) if it is included in every way of negating as many alternatives as possible without contradicting the assertion.

(43) $Kp \approx$ 'the speaker believes that p'

(44) **Obligatory exhaustification of approximators:** Approximators such as *about* must occur in the scope of the exh operator, whose effect cannot be vacuous.

We then can derive the following:

(45) $exh(K$(Lisa has about 50 sheep))
$ALT(p) = \{K$(Lisa has 50_{EXACT} sheep)$\}$
$\rightsquigarrow K$(Lisa has about 50 sheep) $\wedge \neg K$(Lisa has 50_{EXACT} sheep) **correct!**

(46) $exh(K$(Lisa doesn't have about 50 sheep))
$ALT(p) = \{K$(Lisa doesn't have 50_{EXACT} sheep)$\}$
$\rightsquigarrow$ Single alternative is entailed; exh is vacuous. **correct!**

We thus derive both the polarity-based restrictions on the distribution of *about* and the implicature that arises in its felicitous occurrence in positive sentences. However, issues remain:

- **Issue 1: Scope of *exh*:** If the scope of exh and K is reversed, a stronger (unattested?) implicature arises.

(47) $K(exh$(Lisa has about 50 sheep))
$ALT(p) = \{$Lisa has 50_{EXACT} sheep$\}$
$\rightsquigarrow K$(Lisa has about 50 sheep $\wedge \neg$ Lisa has 50_{EXACT} sheep)

- **Too many alternatives:** If more alternatives are included beyond the unmodified one, exh is no longer vacuous where we need it to be.

(48) $exh(K$(Lisa has more than about 50 sheep))
$ALT(p) = \{K$(Lisa has more than 50_{EXACT} sheep)$\}$
$\rightsquigarrow$ Single alternative is entailed; exh is vacuous.

(49) $exh(K$(Lisa doesn't have more than about 50 sheep))
$ALT(p) = \{K$(Lisa doesn't have more than 50_{EXACT} sheep)$\}$

$\rightsquigarrow K$(Lisa doesn't have more than about 50 sheep) $\wedge$
$\neg K$(Lisa doesn't have more than 50_{EXACT} sheep)

– But this does not capture the possible implicature that the speaker believes Lisa to have about 50 sheep.

(50) $exh(K$(Lisa doesn't have more than about 50 sheep))
$ALT(p) = \{K$(Lisa doesn't have more than about n sheep) : $n = \ldots 40, 50, 60, \ldots\} \cup$
$\{K$(Lisa doesn't have more than n_{EXACT} sheep) : $n = \ldots 40, 50, 60, \ldots\}$

$\rightsquigarrow K$(Lisa doesn't have more than about 50 sheep)
$\wedge \neg K$(Lisa doesn't have more than 50_{EXACT} sheep)
$\wedge \neg K$(Lisa doesn't have more than about 40 sheep)

(51) $exh(K$(Lisa has more than about 50 sheep))
$ALT(p) = \{K$(Lisa has more than about n sheep) : $n = \ldots 40, 50, 60, \ldots\} \cup$
$\{K$(Lisa has more than n_{EXACT} sheep) : $n = \ldots 40, 50, 60, \ldots\}$

$\rightsquigarrow exh$ is no longer vacuous! Sentence is grammatical.

To make this work, it appears necessary to specify that exh is only obligatory with respect to the unmodified alternative. Essentially, we need to incorporate a preference for structural simplicity.

5 Extensions and issues

5.1 *Much*

As evidence that the present framework has the potential to be extended to other cases of attenuating polarity items, we consider the case of *much*, which exhibits a variable pattern of polarity sensitivity.

(52) a. *Homer has much money.
b. Homer doesn't have much money.

(53) a. Homer has much more than $100.
b. Homer doesn't have much more than $100.

In both the positive and comparative cases, the *much* sentences compete with the corresponding alternatives without *much*. What differs between them is the interpretation of this unmodified alternative: whereas *Homer has money* suggests he has a significant amount, *Homer has more that $100* has a simple existential interpretation (some amount more than $100). As a result, there is a difference in relative informativity between the *much* sentence and its alternative, which affects the calculation of the 'better than' relation.

(54) ϕ = *Homer has much money* ϕ' = *Homer has money*

$\phi \sim_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi' \succ \phi$

– *Much* version blocked by better simpler alternative

(55) ϕ = *Homer has much more than $100* ϕ' = *Homer has more than $100*

$\phi \succ_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi \sim \phi'$

– *Much* version not blocked

- This case suggests however that we require an iterative approach: first derive the strengthened interpretation of the alternative *Homer has money* and then compare it to the original sentence.

5.2 *Very*

As currently formulated, the definition of informativity in (16) may be too strong, as it appears to block the occurrence of *very* with relative gradable adjectives in positive sentences (cf. the discussion of implicatures and borderline contradictions in Section 2, which was a motivation for the present approach).

(56) ϕ = *Marge is very tall* ϕ' = *Marge is tall*

$\phi \sim_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi' \succ \phi$

There are a number of possible approaches to resolving this issue, that is, accounting for the felicity of positive *very*:

- Simplicity isn't involved in the comparison of *Adj* and *very Adj*; that is, what distinguishes the class of attenuating polarity items is that the 'better than' relation factors in simplicity.
- Leffel et al.'s constraint against implicatures creating borderline contradictions is actually a stronger restriction; there is in fact a sufficient difference in informativity between *Adj* and *very Adj* to allow the latter to surface.
- Some other factor favors the *very* alternative (e.g. politeness).

Note that cross-linguistically, modifiers with the meaning 'very' are frequently NPIs (cf. Matsui 2011 on Japanese *amari* and Hoeksema and Rullmann 2000 on Dutch *bijster*). We might suspect that *very* has the potential to become a polarity item.

6 Conclusions

- In this paper, I have proposed an analysis of the polarity sensitivity of approximators, couched in a neo-Gricean theory of alternatives and assertions.
- The crucial novel components of the present account are the following:
  - The 'better than' relation between alternatives must explicitly factor in a preference for simplicity – at least in the case of attenuating polarity items.
  - Relative informativity must be construed as 'clearly' or 'significantly' more informative than.
- There appears to be potential to extend the present account to other members of the attenuating class, including those with variable polarity sensitivity.
- The same intuitions can be formalized in a grammatical theory of implicatures and polarity, though I have suggested that the present pragmatic approach has some advantages.

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